

EXPERIMENTAL IDENTIFICATION OF A STRUCTURE WITH ACTIVE VIBRATION CANCELLING

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This article describes the process used for the experimental identification and mathematical modeling of an active structure with bonded piezoelectric transducers. A simplified laboratory model is introduced, which is representing an underdamped controlled physical structure, such as a helicopter rotor blade or solar panels and antenna masts in space. A twelfth order state-space model of the physical system is obtained through subspace methods utilizing frequency domain measurement data. The resulting state-space structure captures vibrational behavior under five transversal bending eigenfrequencies of the seven included in the bandwidth of interest. The mathematical model is further utilized in an efficient predictive controller with constraint handling capability.

Keywords: active structure, smart material, vibration cancelling, vibration damping, piezoelectric, predictive control

Príspevok prináša informácie z experimentálnej identifikácie a matematického modelovania aktívnych štruktúr s piezoelektrickými prevodníkmi. Článok predstavuje zjednodušený fyzikálny model, ktorý znázorňuje podtlmený, riadený systém ako napríklad vrtuľníkové lopaty alebo solárne panely a antény vo vesmíre. Dvanásť rádoový stavový model je získaný použitím podpriestorovej identifikácie a meraní vo frekvenčnej oblasti. Výsledný stavový model zachytí vibračné správanie systému pod piatimi priečnymi vlastnými frekvenciami v danej šírke pásma. Matematický model bude ďalej využitý vo výpočtovo výkonnom prediktívnom riadiacom algoritmu, so schopnosťou znázorňovania procesných obmedzení.

Kľúčové slová: aktívna štruktúra, smart materiál, tlmenie vibrácie, piezoelektricitá, prediktívne riadenie

1. INTRODUCTION

1.1. Motivation of the work.

Vibration is present in countless real life applications, and most of the time it is a highly undesirable phenomenon. Unwanted vibration may decrease product performance, cause economic or safety problems. All physical systems have some inherent physical damping, but in some cases the level may not be satisfactory. To increase energy dissipation one may apply vibration attenuation techniques. This way the response of a structure driven at resonant frequencies may be greatly decreased. One may choose passive, semi-active and active approaches.

Due to several issues, passive treatments are not always viable. Active vibration damping can be an attractive alternative, which employs actuators to utilize external force effects on the

system in question. These actuators are driven by control systems, which gain feedback from one or several sensors. Such active structures are highly integrated and may be regarded as one complex mechatronic unit.

There is one very important and often overlooked aspect of active vibration attenuation systems: the control algorithm. The obvious and simplest choices are well investigated. The basic controllers implemented often do not provide the necessary performance; in some cases stability is also questionable. Input and output constraints may be required because of actuator limitations, safety or economic considerations. Currently the only control technique, which can deal with constraints and their effect on future control actions, is model predictive control (MPC).

Model predictive control employs a state-space mathematical model of the structure to determine the future optimal control signals. In order to ensure the feasibility of the constraints and the correct functionality of the controller itself, it is essential to obtain a good approximation of the physical system's behavior.

1.2. Problem statement.

One may think of a very simple active vibration damping example, which illustrates numerous real-life applications. A cantilever beam clamped at one end having the other free may represent a helicopter rotor blade. The vibration of a rotor blade in flight is undesirable since decreases performance and increases fuel consumption. Another obvious application is the vibration damping of large space structures. Such structures may be antenna masts, solar panels or the remote manipulation arm of the Space Shuttle.

A laboratory device was completed to model the above defined real-life situation. In order to implement and test a conceptual efficient (but somewhat sub-optimal) predictive controller [1], one must obtain a proper mathematical model of the system behavior in the given bandwidth. The initial aim is to control 5 transversal bending modes of the beam, which for our current configuration approximates a 500Hz bandwidth. The reason for including higher modes in the model is to justify the usage of a high sampling rate application, thus requiring a computationally efficient MPC controller. A 500Hz bandwidth requires at least a sampling rate of 5000Hz. (0.0002 seconds.)

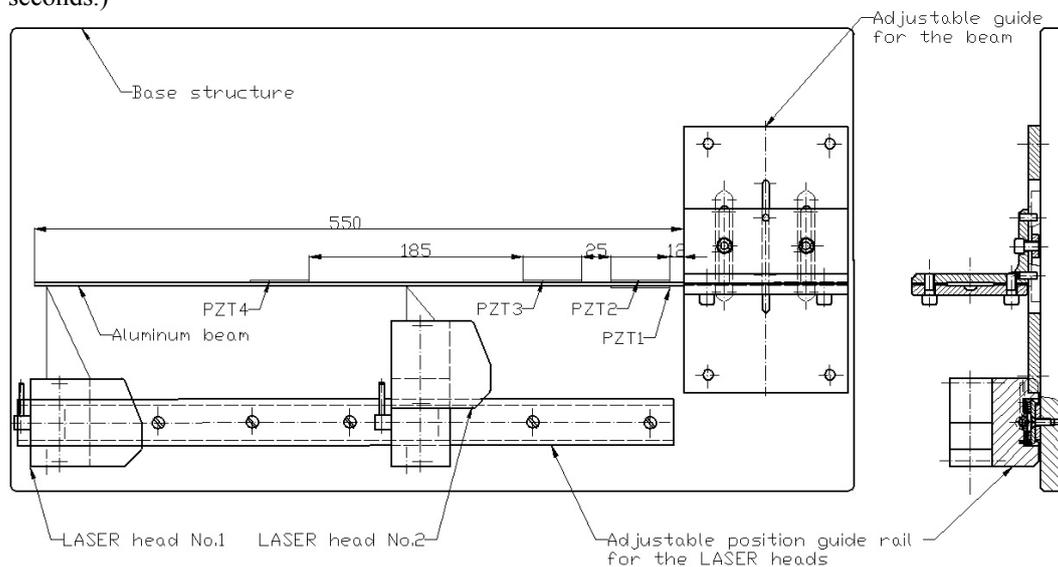


Fig. 1. Laboratory model hardware configuration.

2. EXPERIMENTAL IDENTIFICATION OF THE ACTIVE SYSTEM

2.1. Description of the physical model and hardware

The laboratory model in question consists of a commercially available 99.5% pure aluminum beam, with one end clamped and the other left to vibrate freely. The beam dimensions are $550 \times 40 \times 3$ mm and its material designation is EN AW 1050A.

This beam has several MIDÉ QP16 piezoelectric transducers bonded on its surface with high strength epoxy glue. (See Figure 1.; marked as PZT1 to PZT4) To control the beam tip displacement the transducers closer to the clamped end (PZT1 and PZT2) are employed as actuators. Proximity of these transducers to the fixed end ensures maximal possible bending moment. The input of the actuator mode piezoelectric wafers is fed through a MIDÉ EL-1225 power amplifier. The voltage signal to the actuators in question is of opposite polarity. The initial control system tests assume a directly measured beam-tip displacement signal. Future implementations may employ a feedback signal based on a measurement model and the electrical signals originating from sensor mode piezoelectric transducers.

The direct beam tip displacement is measured through the use of high precision industrial LASER triangulation devices of the type KEYENCE LK - G 82 complete with a proprietary processing and control unit and settings software.

The signal coming from the distance measuring devices and the one fed to the power amplifier is processed by a high sampling rate, 16bit National Instruments PCI-6030E measuring card. This measuring board is located in a "target" computer running the Mathworks proprietary real-time control environment xPC Target. The control and identification software, models and other algorithms are loaded to the machine running xPC Target from the "host" computer via Ethernet. The host computer runs the Matlab / Simulink suite and the settings and control environment for the LASER triangulation devices.

The complete laboratory setup is shown on Figure 2. with all the auxiliary equipment explicitly not mentioned in the description above.



Fig. 2. Complete laboratory setup

2.2 Experiment design

Prior to the design of the beam, series of finite element method (FEM) simulations were completed to determine the ideal shape and dimensions of the vibrating cantilever. Emphasis was placed on the suitable distribution of eigenfrequencies within the bandwidth of interest. The final beam turned out to have seven eigenfrequencies and modes under 500 Hz. Five of them are transversal vibrational modes, the third eigenmode shows a sideways bending movement and the fifth one is a twisting mode. The comparison between the measured and FEM simulated eigenmodes in Table 1. show a good agreement.

Table 1.: FEM and experimental eigenfrequency comparison

Mode No.	1.	2.	3.	4.	5.
FEM Frequency (Hz)	8.2	50.6	139.8	271.9	449.2
Experimental Frequency (Hz)	8.2	50.7	140.9	275.5	455.1

The FEM simulation also provides the possibility to retrieve a mathematical model from its simulation results, particularly from the harmonic ones. [2] Although this would be a viable approach, the properties of the FEM models prohibit obtaining a precise enough model for the needs of MPC control. Some of the physical properties of the system are not exactly known, and cannot be measured within the means of our laboratory; therefore the results may be skewed. Also assumed the Rayleigh damping will distort the amplitude levels especially at higher frequencies.

Developing an explicit solution to the problem would be a favorable choice. Despite of the complexity of this approach, several works attempt to utilize it. [3-4]

After reviewing the possibilities a purely experimental approach was chosen in the frequency domain. Initial plans included the explicit measurement of amplitude and signal phase-lag at discrete frequency points when subjected to sinusoidal excitation in the 0 – 500Hz bandwidth. This data would be sufficient to manipulate with when obtaining the state - space model. In addition to measuring the amplitude levels when PZT1 and PZT2 subjected to a harmonic signal a phase lag / lead information was needed. As several attempts showed, the development of a real-time, online phase lag detection algorithm is not trivial. Three different approaches were attempted:

- Zero crossing detection
- Hilbert phase lag detection
- Yunus phase lag detection

The zero crossing detection method involves the measurement of discrete time points when the measured signal crosses the time axis. These times are then compared to the reference signal and a phase lag or lead angle is directly calculated. Hilbert phase lag detection involves an online transformation using a Hilbert filter, to create a complex helical sequence – sometimes called an analytical signal. Phase information is subtracted then from this signal.

Yunus phase lag detection starts from the well known formula for the multiplication of two sinusoidal signals:

$$\sin \theta_1 \sin \theta_2 = \frac{1}{2} [\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)] \quad (1.)$$

where one sinusoidal signal describes the reference signal, the other the measured signal. By the multiplication of these two signals, one may extract the phase information.

Although all three proposed methods should theoretically provide a precise on-line phase value, when subjected to a noisy, shifted or trending signal neither of them showed satisfactory results. Considerable amount of on-line statistical processing did not provide a significant improvement in precision or reliability. In addition to the problems arising from the nature of the measured signal, inherent properties of computational systems used.

Zero crossing detection was entirely unreliable, when subjected to frequencies above ~20Hz. (0.0002 sampling considered) Yunus phase detection provided a good match up to a certain point in the frequency scale (~150Hz) – then it too became imprecise or failed completely.

Although the method utilizing Hilbert transformation was the most promising, the idea of measuring amplitude and phase data in discrete points on-line was abandoned due to the above mentioned issues with reliability and precision. Instead a method using a time domain signal was chosen, with the subsequent transformation of the measured data into the frequency domain for the purposes of practical identification and mathematical modeling.

2.3 Practical measurements

The driving force in the piezoelectric transducers PZT1 and PZT2 was chosen to be a chirp signal. The excitation function was generated in a Matlab / Simulink scheme, where the amplitude of the signal going into the measuring card was $\pm 5V$. This signal was then fed through the power amplifiers with a $20 \times$ gain to reach the maximal possible voltage level on the actuators: $\pm 100V$ RMS. The high voltage level is due to the need to minimize signal to noise ratio in the measurement signals, especially in between resonant frequencies.

The LASER triangulation sensors have a limited precision for a given deflection span, therefore the bandwidth from 0 to 500 Hz was divided to parts accordingly. At each partial measurement, the LASER properties were re-set on the basis of the expected maximal deflection and frequency. To improve precision, the amplitude / voltage ratio was modified. Digital low-pass filtering was enabled to improve precision. The measurements were carried out according to the data presented in Tab. 2.

Table 2.: Measurement settings

Pass No.:	1.	2.	3.	4.	5.
Frequency span (Hz):	0.01-7	7-9	9-45	45-55	55-500
Amplitude gain (mm/V)	0.1	1.5	0.2	0.5	0.25
Low pass filter cutoff: (Hz)	10	10	100	100	1000
No. of samples [$\times 1e6$ samp.]	1.5	3	1	3	5

Sampling rate was determined to be 0.0002 seconds, which is 5000 Hz. This would allow sufficient resolution even at the higher end of the required bandwidth. To reach the resonant amplitudes at the eigenfrequencies, one had to leave sufficient time for the measurement. If the chirp signal passes through the resonances too fast, resulting maximal vibrational amplitudes will be lower than expected.

This brings an additional factor in consideration – that is the high number of measurement points to be stored in real-time. xPC target system provides the capability to use file scopes, with a storage capacity only limited by the target machines RAM. The computer used for the measurements had a 300MB RAM, which enabled to store approximately 14 million samples along with the time series data. The resulting data file was retrieved and transcoded into a Matlab usable format using xPC target specific Matlab commands. [5]

The partial measurement files were combined into one result file using Matlab. The resulting raw time series data contained 13.5 million samples. Manipulation with a data vector of this size has been difficult on a personal computer conforming to today's standard specifications.

2. 4 System identification procedure

The raw data file was loaded into the Matlab System Identification Toolbox. The post-processing of the measurements included detrending and removing signal means. To convert the time series measurements to frequency domain, a fast Fourier transformation (FFT) was performed on the measured data set, up to the half of the sampling bandwidth: that is 2500Hz. This data file was then filtered using a low-pass band filter to cut off unnecessary frequencies and to reduce the amount of working data.

Since the amount of data points in the working file still prohibited the practical use of identification routines, the frequency response and the spectrum was estimated using spectral analysis with frequency-dependent resolution returning. The estimation procedure has been performed with a logarithmic resolution of 2000 frequencies, from 1 to 500Hz. The resulting processed measurement was suitable for direct identification.

The considered model predictive controller requires the use of a state-space mathematical model of the physical system. After comparing singular values and taking into consideration the required bandwidth, a 12th order model has been chosen. The identification routine utilized a subspace iteration method, implemented as default in the toolbox [6].

3. TESTING AND VERIFICATION OF THE RESULTS

3. 1 Comparison of the model and the physical system

Ideally the comparison of the simulation results and real-life measured data would be performed in the frequency domain. The particular method of obtaining the frequency domain measurements unfortunately prohibits the direct comparison of the results. The reason for this is the size of the raw frequency domain measurement file: comparison on computers conforming today's standards and using the System Identification Toolbox is not viable.

Comparison of the partial bandwidth measurements is shown on Figure 3a. In this case the response of a mere second order system is compared with the simulation results up to a partial bandwidth of ~20Hz. Change in the resolution settings of the LASER system and different measurement properties can be clearly differentiated in the provided figure.

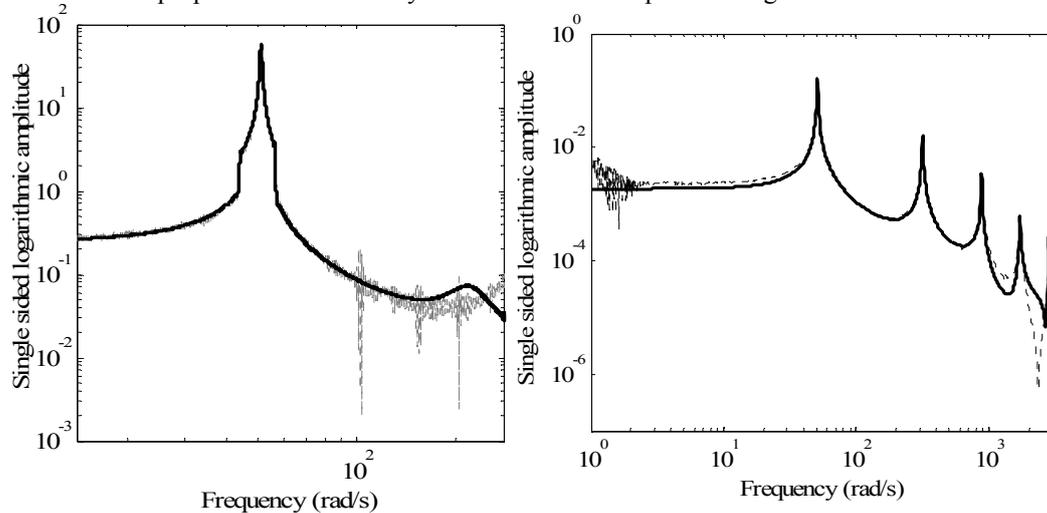


Figure 3a and 3b.: Comparison in partial bandwidth and spectral estimate

Comparison in the frequency domain is practically viable by using the frequency response and spectrum estimate. The System Identification Toolbox then also provides a numerical indicator of precision – a percentage match between the model and measurement data. One has to take into account also the planned practical implementation and use of the state-space model to validate the results. In this case the match of the model in the proximity of the first resonant frequency is more important than the general correspondence in the whole bandwidth. In the process of fine-tuning the identification algorithm, fitting the model to measurement data within the high frequency portions of the signal was of less importance. Graphical results of this comparison in Figure 3b.

The comparison of the measurement results in the time domain is not a good indicator of the model accuracy. The main reason for this is the fact that in static mode the beam tip deflection is minimal, only measurable in tenths of millimeters – the practical use will utilize a dynamic excitation of the piezoelectric transducers anyways. Outside disturbances have a substantial effect on the measurements. The physical device is also placed in an environment, where the effect of road traffic or people moving within the room is clearly indicated in the movement of the beam tip.

The direct comparison of the model response and the real system measurements to a pulse signal is shown on Figure 4. Disturbances are present during the course of measurements, and can be clearly differentiated after the settling time of approximately 20 seconds.

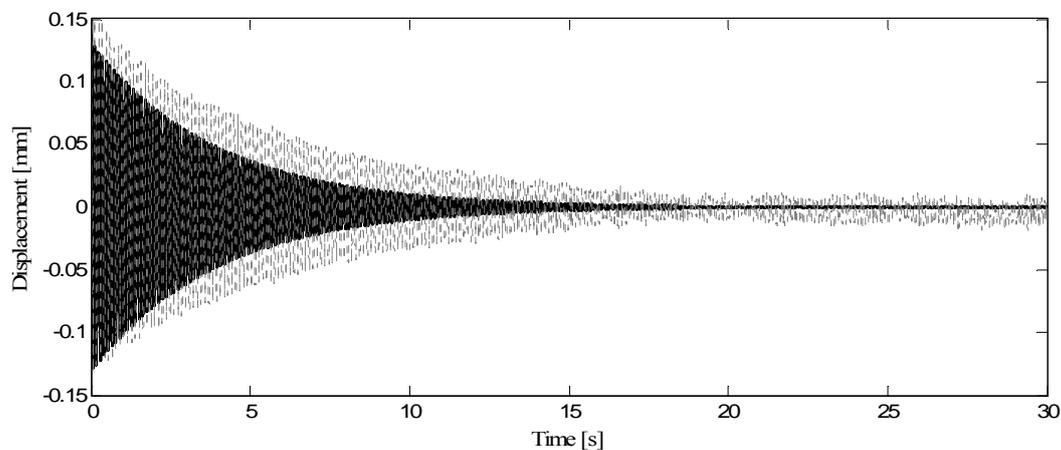


Figure 4. Comparison of the model and physical system step response

3. 2 Practical use of the state-space model

At the time of preparation of this paper, the predictive controller which is planned to be used as the vibration damping algorithm is only in its simulation testing phase. The C language implementation necessary for the use under xPC Target in real time is yet to be completed [7].

For this reason a simple linear quadratic (LQ) controller was assumed. The controller matrix gain has been computed using “dlqr” routine which utilizes an algorithm to compute the discrete optimal state feedback controller gain using the A and B matrices of the state-space system and penalization matrices. A Kalman observer was used to acquire the system states from the measured past output signals.

Figure 3. compares the free and the LQ controlled response to an initial beam tip displacement of 5mm. As it is clear from the figure, the controller response shows a significant

improvement in the damping properties of the system. While the LQ controlled beam tip vibration settles in approximately a second, the uncontrolled movement continues up to a minute.

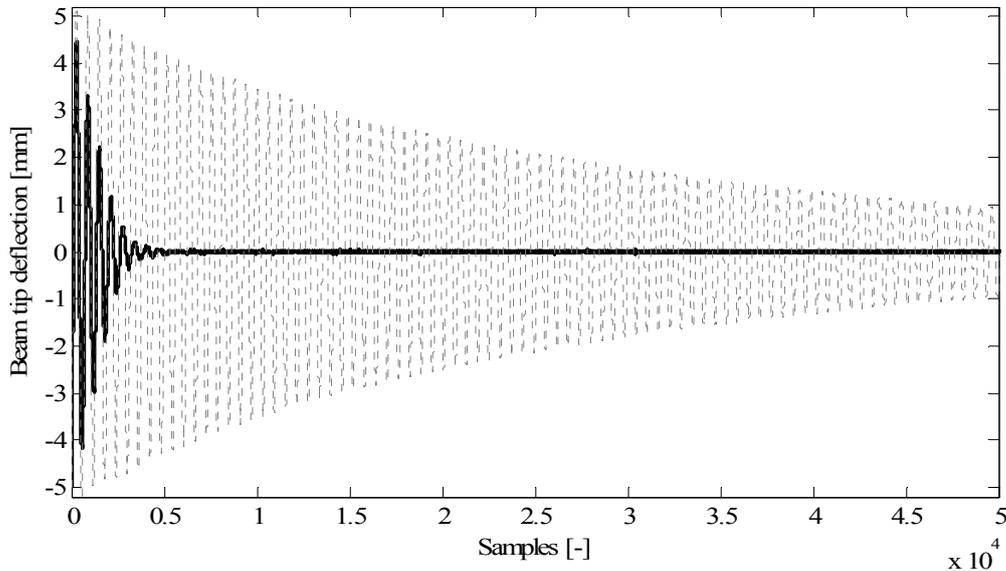


Figure 5.: Free compared to LQ controlled response to initial 5 mm displacement.

4. CONCLUSION

The practical aspects of the experimental identification of an active system with vibration attenuation have been introduced in this paper. After the problem statement the laboratory device has been introduced, along with the used measurement process and the identification procedure. The measurement process showed difficulty with working with and evaluating large datasets. A validation of the results has been performed focusing on practical trials.

Comparison of the measurement data both in frequency domain and especially in the time domain did not provide a good indication of model quality. Rather the results of the practical model validation show, that the state-space mathematical model of the laboratory device well represents the system behavior. Outcome of the experiments with an LQ optimal controller indicate, that the state-space model of the vibrating beam tip will be also suitable to use with a predictive controller algorithm.

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